AIAA JOURNAL
Vol. 44, No. 8, August 2006

Combined Direct/Adjoint Reduced-Order Approximations for Design-Oriented Structural–Acoustics

E. Livne* and L. Coskuner†
University of Washington, Seattle, Washington 98195-2400

The accuracy of modal-order reduction methods in structural dynamics and structural acoustics depends on the particular problem analyzed and particular modal bases selected. Modal-order reduction methods can become quite inaccurate when sensitivities of response functions with respect to design variables are required. Past research in linear aerodynamics and structural dynamics had shown how direct and adjoint solutions can be combined to yield reduced-order analysis and sensitivity approximations of the problems at hand with second-order accuracy. Efforts are reported to extend the methodology to structural acoustic problems of fluid-filled cavities enclosed by flexible walls. In this case, stress and pressure response to excitation has to be evaluated over a range of frequencies. Improved and more consistent accuracy is reported over a range of frequencies with the second-order method when accuracy of both approximate direct and approximate adjoint solutions is comparable. The second-order method cannot improve accuracy of first-order methods in general when one of the approximate first-order solutions is highly inaccurate.

Nomenclature

$[A]$	=	coefficient matrix for a general linear system of equations
$\{b\}$	=	general right-hand side of a linear system of equations
$\{c\}$	=	vector of weights for calculating the scalar response of a linear system
$\{F\}$	=	vector of input forces
g	=	structural damping coefficient
$[K]$	=	coupled structural–acoustic stiffness matrix
$[M]$	=	coupled structural–acoustic mass matrix
p	=	pressure
$[R]$	=	structural–acoustic coupling matrix
$\{u(j\omega)\}$	=	direct solution of the frequency-domain structural–acoustic problem
$\{x\}$	=	direct solution of a general linear problem
$\tilde{x}$	=	approximate direct solution
y	=	scalar response of a linear system
$\tilde{y}$	=	approximate response
$\{\eta\}$	=	adjoint solution of a general linear problem
$\{\eta(j\omega)\}$	=	adjoint solution of the frequency-domain structural–acoustic problem
$\tilde{\eta}$	=	approximate adjoint solution
ρ	=	fluid density
$[\Phi_L]$	=	reduced basis matrix for premultiplying the direct problem (can be a set of left eigenvectors or Ritz vectors)
$[\Phi_R]$	=	reduced basis matrix for the direct problem (can be a set of right eigenvectors)
$[\Psi_L]$	=	reduced basis matrix for premultiplying the adjoint problem (can be a set of left eigenvectors or Ritz vectors)
$[\Psi_R]$	=	reduced basis matrix for the adjoint problem (can be a set of right eigenvectors or Ritz vectors)

Subscripts

a	=	acoustic
s	=	structural

I. Introduction

DETAILED large-scale structural–acoustic finite element mathematical models in the time or frequency domains[1–11] play an important role in the analysis of complex dynamic systems and in any effort to develop related effective design optimization tools. Applications of particular importance include structural acoustics of aircraft, ground vehicles, and submarines; vibroacoustics of missile payloads and sensitive electrooptic equipment; and structural acoustics of buildings. Comprehensive design models of such systems should be capable of capturing both global-scale behavior, such as overall deformation and pressure distribution, and local behavior, such as stress and stress concentration for fatigue and damage tolerance analysis, local buckling of structural subcomponents, etc.

With the most advanced computational technology available today, structural–acoustic systems still present a significant analysis and design challenge. Mathematical models containing hundreds of thousands of degrees of freedom, and a large number of frequencies, where frequency response has to be evaluated (of the order hundreds to thousands), lead to computationally intensive problems.[12] In the time domain, thousands of time steps are required for simulation. Even when computer parallelization on hundreds of CPUs is used for the frequency-response problem, where each processor handles the detailed dynamic response problem at one frequency only, this is still a demanding problem, given the size of the dynamic response problem at a single frequency. When behavior sensitivities are required for driving design optimization of the systems modeled, the resulting computational time and storage requirements are formidable. Many large-scale structural acoustic system optimization and synthesis problems of the classes listed are still intractable.

Model-order reduction techniques and computationally efficient approximations are used for design and design optimization to replace large-scale detailed models, which, when used repetitively, lead to prohibitive computational costs.[13,14] A variety of general standard behavior approximations for optimization are available today, including, in the simplest case, first-order Taylor-series-based direct and reciprocal models. These Taylor series approximations are constructed using analysis and sensitivity information at selected

Presented as Paper 2004-1643 at the AIAA/ASME/ASCE/AHS/ASC 45th Structures, Structural Dynamics, and Materials Conference, Palm Springs, CA, 19–22 April 2004; received 20 August 2004; revision received 17 December 2005; accepted for publication 29 January 2006.

*Professor, Department of Aeronautics and Astronautics. Associate Fellow AIAA.
†Graduate Student, Department of Aeronautics and Astronautics.

points in the design space, and because of their local nature, acceptable accuracy can be expected only in the proximity of these reference design points.

As part of an effort to improve accuracy of approximation methods based on first-order sensitivity information, a new method, based on the solution of combined approximate direct and adjoint problems, has been found[15–17] to lead to accurate approximations in exploratory studies of structural static and dynamic stresses in airframe structures (time domain and frequency domain), as well as unsteady (frequency domain) aerodynamic forces on wings. These reduced-order approximations were shown to posses second-order accuracy,[18] similar to the second-order accuracy of Rayleigh quotient approximations widely used in structural optimization.[19]

The second-order approximation (SOA) method is based on the replacement of full-order dynamic problems (of the order of hundreds of thousands) by groups of small low-order approximate problems (of the order of tens, and, in many cases, even less than 10×10). Response and sensitivities, then, are calculated by the small, reduced-order, models with second-order accuracy.

II. Coupled Structural–Acoustic Problem

The acoustic equations of motion for an acoustic enclosure coupled to a flexible structure are

$$[[M_a]\{\ddot{p}\} + [K_a]\{p\} - \rho_0[R_{as}]\{\ddot{u}_s\}] = \{0\} \tag{1}$$

For the equations of the structural part, we have (assuming viscous damping and a applied external forces)

$$\Big[[M_s]\{\ddot{u}_s\} + [C_s]\{\dot{u}_s\} + [K_s]\{u_s\} - [R_{as}]^T\{p\}\Big] = \{F\} \tag{2}$$

The matrices $[M_a]$ and $[K_a]$ are acoustic mass and stiffness matrices, respectively. The matrices $[M_s]$, $[K_s]$, and $[C_s]$ are structural mass, stiffness, and damping matrices, respectively. $[R_{as}]$ is the structural–acoustic coupling matrix, and $\{u_s(t)\}$, $\{p(t)\}$ and $\{F(t)\}$ are vectors of structural displacements, perturbation pressures, and structural excitation forces, respectively. Here ρ_0 is the density of the fluid.

When Eqs. (1) and (2) are combined, the coupled structural–acoustic equations are

$$\begin{bmatrix} M_s & 0 \\ -\rho_0 R_{as} 0 & M_a \end{bmatrix} \begin{Bmatrix} \ddot{u}_s \\ \ddot{p} \end{Bmatrix} + \begin{bmatrix} C_s & 0 \\ 0 & 0 \end{bmatrix} \begin{Bmatrix} \dot{u}_s \\ \dot{p} \end{Bmatrix} + \begin{bmatrix} K_s & -R_{as}^T \\ 0 & K_a \end{bmatrix} \begin{Bmatrix} u_s \\ p \end{Bmatrix} = \begin{Bmatrix} F \\ 0 \end{Bmatrix} \tag{3}$$

III. Full-Order and Reduced-Order Models

Fourier transformation of the dynamic equations (3) for a structural acoustic system in the frequency domain leads to

$$[-\omega^2[M] + j\omega[C] + (1 + jg)[K]]\{u(j\omega)\} = \{F(j\omega)\} \tag{4}$$

where in this formulation $[C]$ is the coupled structural–acoustic viscous damping matrix and $[K]$ is the coupled structural–acoustic stiffness matrix (3). A hysteretic structural damping force $jg[K]\{u\}$ has been added to the equations. A scalar response such as stress, displacement, or pressure at a point can be obtained from the solution of Eq. (4) by

$$y(j\omega) = \{c\}^*\{u(j\omega)\} = \{\eta(j\omega)\}^*\{F(j\omega)\} \tag{5}$$

where the corresponding full-order adjoint equations are

$$[-\omega^2[M] + j\omega[C] + (1 + jg))[K]]^*\{\eta(j\omega)\} = \{c\} \tag{6}$$

The superscript asterisk denotes the transpose complex conjugate of a complex matrix. The coupled structural–acoustic mass, damping, and stiffness matrices are in general real and nonsymmetric. The adjoint problem is then

$$[-\omega^2[M]^T - j\omega[C]^T + (1 - jg)[K]^T]\{\eta(j\omega)\} = \{c\} \tag{7}$$

The vector $\{c\}$, defining the output, is usually frequency independent when the response y is displacement, stress, or pressure.

The term full-order is used here to describe the system of equations corresponding to a detailed finite element model with a fine mesh and many degrees of freedom. In the mode displacement (MD) method, a reduced-basis matrix $[\Phi_R]$ is used to reduce the order of the full-order problem by approximating the full-order response as some linear combination of a small number of modal base vectors:

$$\{u_{\mathrm{MD}}(j\omega\} \approx [\Phi_R]\{q(j\omega)\} \tag{8}$$

with the vector $\{q(j\omega)\}$ containing the generalized displacements and pressure perturbations.

If the full-order equation as given in Eqs. (3) and (4) is written as

$$[A(j\omega)]\{u(j\omega)\} = \{F(j\omega)\} \tag{9}$$

Then $[A]$ is reduced using approximation (8) and after premultiplication by $[\Phi_L]^*$ such that

$$[A_\Phi] = [\Phi_L]^*[A][\Phi_R] \tag{10}$$

and the generalized input force is

$$\{F_\Phi(j\omega)\} = [\Phi_L]^*\{F(j\omega)\} \tag{11}$$

We now have a reduced-order direct problem:

$$[A_\Phi]\{q(j\omega)\} = \{F_\Phi(j\omega)\} \tag{12}$$

Similarly, a reduced-order adjoint frequency-response problem can be obtained by using another reduced-basis matrix $[\Psi_R]$ to approximate the full-order adjoint solution by some superposition of the basis vectors of $[\Psi]$. The generalized adjoint displacements are given by the vector $\{r(j\omega)\}$, such that

$$\{\eta(j\omega\} \approx [\Psi_R]\{r(j\omega)\} \tag{13}$$

When the full-order equations (6) and (7) are written in the form

$$[A]^*\{\eta(j\omega)\} = \{c\} \tag{14}$$

then $[A]^*$ is reduced using the basis matrices $[\Psi_L]$ and $[\Psi_R]$ as follows:

$$\Big[A_\Psi^*\Big] = [\Psi_L]^*[A^*][\Psi_R] \tag{15}$$

and the corresponding reduced $\{c\}$ vector is given by

$$\{c_\Psi\} = [\Psi_L]^*\{c\} \tag{16}$$

The reduced-order adjoint problem is

$$[A_\Psi]^*\{r(j\omega)\} = \{c_\Psi(j\omega)\} \tag{17}$$

The selection of reduced bases $[\Phi_L]$, $[\Phi_R]$ and $[\Psi_L]$, $[\Psi_R]$ is not straightforward and depends on the spatial distribution of excitation forces, the weights used to determine particular responses (the vector $\{c\}$), as well as the frequency of oscillation. Utilization of natural modes of the structure, fictitious mass modes, Ritz vectors, or alternative subcomponent modes has been studied extensively over the years in the context of structural dynamics.[20–29] In the general structural–acoustics case discussed here, mode shape (reduced-basis) matrices are allowed to be complex.

IV. Second-Order Approximation Using Combined Approximate Direct and Adjoint Solutions

A direct solution $\{x\}$ of a linear problem is obtained by solving the matrix equation

$$[A]\{x\} = \{b\} \tag{18}$$

A particular scalar response y can be found using either the direct or the adjoint solution:

$$y = \{c\}^*\{x\} = \{c\}^*[A]^{-1}\{b\} = \{\eta\}^*\{b\} \tag{19}$$

where a complementary adjoint problem is given by

$$[A]^*\{\eta\} = \{c\} \tag{20}$$

Suppose now that some approximate solutions $\{\tilde{x}\}$, $\{\tilde{\eta}\}$ of the direct and adjoint problems are available [through approximation (8) and Eq. (12) or approximation (13) and Eq. (17)]: $\{\tilde{x}\}$, $\{\tilde{\eta}\}$, $\tilde{y}$. The corresponding errors due to approximation are then

$$\{\delta x\} = \{x\} - \{\tilde{x}\}, \qquad \{\delta\eta\} = \{\eta\} - \{\tilde{\eta}\}, \qquad \delta y = y - \tilde{y} \tag{21}$$

An SOA of y is, then, created[15–17] by using the approximate direct and adjoint solutions and the full-order exact matrix $[A]$ and vectors $\{b\}$ and $\{c\}$:

$$\tilde{y} = \{c\}^T\{\tilde{x}\} + \{\tilde{\eta}\}^T\{b\} - \{\tilde{\eta}\}^T[A]\{\tilde{x}\} \tag{22}$$

Because

$$\{\tilde{x}\} = \{x\} - \{\delta x\}, \qquad \{\tilde{\eta}\} = \{\eta\} - \{\delta\eta\}, \qquad \tilde{y} = y - \delta y \tag{23}$$

substitution into Eq. (23) leads to a second-order error:

$$\delta y = -\delta\eta^T A \delta x \tag{24}$$

Differentiation of Eq. (22) with respect to a design variable (DV) leads to the SOA for the sensitivities. The sensitivities of the approximate direct and adjoint solutions, $\partial\{\tilde{x}\}/\partial\mathrm{DV}$and $\partial\tilde{\eta}/\partial\mathrm{DV}$, are used together with the sensitivities of the full-order vectors and matrices $\partial\{b\}/\partial\mathrm{DV}$, $\partial\{c\}/\partial\mathrm{DV}$ and $\partial[A]/\partial\mathrm{DV}$ to write the SOA of the sensitivity as

$$\frac{\partial\tilde{y}}{\partial\mathrm{DV}} = \frac{\partial}{\partial\mathrm{DV}}(\{c\}^*\{\tilde{x}\} + \{\tilde{\eta}\}^*\{b\} - \{\eta\}^*[A]\{\tilde{x}\}) \tag{25}$$

Upon expanding,

$$\begin{aligned}\frac{\partial\tilde{y}}{\partial\mathrm{DV}} &= \frac{\partial\{c\}^*}{\partial\mathrm{DV}}\{\tilde{x}\} + \{c\}^*\frac{\partial\{\tilde{x}\}}{\partial\mathrm{DV}} + \frac{\partial\{\tilde{\eta}\}^*}{\partial\mathrm{DV}}\{b\} + \{\tilde{\eta}\}^*\frac{\partial\{b\}}{\partial\mathrm{DV}}\\ &\quad -\frac{\partial\{\tilde{\eta}\}^*}{\partial\mathrm{DV}}[A]\{\tilde{x}\} - \{\tilde{\eta}\}^*\frac{\partial\,[A]}{\partial\mathrm{DV}}\{\tilde{x}\} - \{\tilde{\eta}\}^*[A]\frac{\partial\{\tilde{x}\}}{\partial\mathrm{DV}}\end{aligned} \tag{26}$$

and after collecting terms,

$$\begin{aligned}\frac{\partial\tilde{y}}{\partial\mathrm{DV}} &= (\{c\}^* - \{\tilde{\eta}\}^*[A])\frac{\partial\{\tilde{x}\}}{\partial\mathrm{DV}} + \frac{\partial\{\tilde{\eta}\}^*}{\partial\mathrm{DV}}(\{b\} - [A]\{\tilde{x}\})\\ &\quad - \{\tilde{\eta}\}^*\frac{\partial[A]}{\partial\mathrm{DV}}\{\tilde{x}\} + \frac{\partial\{c\}^*}{\partial\mathrm{DV}}\{\tilde{x}\} + \{\tilde{\eta}\}^*\frac{\partial\{b\}}{\partial\mathrm{DV}}\end{aligned} \tag{27}$$

The term $(\{b\} - [A]\{\tilde{x}\})$ will be recognized as the error vector in the full-order equation (18) when the approximate direct solution replaces the exact solution. Similarly, the term $(\{c\}^* - \{\tilde{\eta}\}^*[A])$ is the error vector in the full-order adjoint equation (19) in the presence of an approximate adjoint solution instead of the exact adjoint solution. If both the approximate direct and the approximate adjoint solutions are accurate, a simplified equation for the sensitivity of the SOA can be obtained:

$$\frac{\partial\tilde{y}}{\partial\mathrm{DV}} = \frac{\partial\{c\}^*}{\partial\mathrm{DV}}\{\tilde{x}\} + \{\tilde{\eta}\}^*\frac{\partial\{b\}}{\partial\mathrm{DV}} - \{\tilde{\eta}\}^*\frac{\partial[A]}{\partial\mathrm{DV}}\{\tilde{x}\} \tag{28}$$

Because in many practical design optimization problems the inputs to the system ($\{b\}$) do not change, and in many cases (such as when displacement or acoustic pressure constraints are involved) the vector $\{c\}$ is not affected by design changes, Eq. (28) can be further simplified to yield

$$\frac{\partial\tilde{y}}{\partial\mathrm{DV}} = -\{\tilde{\eta}\}^*\frac{\partial[A]}{\partial\mathrm{DV}}\{\tilde{x}\} \tag{29}$$

In the case of stress constraints, where the vector $\{c\}$ depends on DV in the form of element cross-sectional area parameters, a simplified derivative of the second-order approximate behavior output is

$$\frac{\partial\tilde{y}}{\partial\mathrm{DV}} = \frac{\partial\{c\}^*}{\partial\mathrm{DV}}\{\tilde{x}\} - \{\tilde{\eta}\}^*\frac{\partial[A]}{\partial\mathrm{DV}}\{\tilde{x}\} = \left(\frac{\partial\{c\}^*}{\partial\mathrm{DV}} - \{\tilde{\eta}\}^*\frac{\partial[A]}{\partial\mathrm{DV}}\right)\{\tilde{x}\} \tag{30}$$

V. Second-Order, Reduced-Order Approximations for the Structural–Acoustic Problem

Order reduction techniques for the structural acoustics problem are widely used.[9,28,29] In the most common approach, natural modes of the structure are used to reduce the order of the structural system. They are coupled with a reduced-order acoustics model based on superposition of some acoustic modes in a variant of the substructure modal synthesis method of structural dynamics.[20] In a typical design optimization implementation, detailed (full-order) solutions are calculated for a system corresponding to some reference (base) design. Reduced-order approximations based on the reference design are then calculated repeatedly for the system, as it undergoes variation and modification during the optimization process. With the SOA just presented, a number of options may be considered for the creation of approximate analyses and sensitivity analyses. Two first-order approximations are required when the SOA method is used, and these first-order approximation can be any of the following:

First, it may be a fixed-modes order reduction for either the direct or the adjoint models, based on the natural modes of the reference structure [approximation (8) and Eqs. (9–12) and approximation (13) and Eqs. (14–17)]. Variable-modes approximations are possible, too, but are not used here because of the computational cost of obtaining eigenvector derivatives.

When modes of the reference system (complex, in general) are used for order reduction, a low-frequency subset of the right natural modes $[\Phi_R]$ [see Eq. (4)]:

$$[\lambda_i[M] + [K]]\{\phi_i\} = \{0\} \tag{31}$$

is used for the direct problem [Eqs. (10–12)], or a subset of the left complex modes $[\Psi_L]$

$$\left[\lambda_i[M]^T + [K]^T\right]\{\psi_i\} = \{0\} \tag{32}$$

is used for the adjoint problem [Eqs. (15–17)]. Both right and left modes are used to reduce either the direct or adjoint problem. When modes are used for one problem (direct or adjoint), modes of the same structure cannot be used for the other problem (adjoint or direct). It is straightforward to show that when the same modes are used for both direct and adjoint reduced-order problems, the SOA loses its second-order accuracy and becomes first order. Because the structural–acoustic mass and stiffness matrices in the formulation used here are nonsymmetric, the mode shapes are in general complex.

Second, it may be a fixed-modes order reduction for either the direct or the adjoint models based on modes of the reference structure when it is modified by fictitious masses (FM) to create a desirable set of modes.[27] When modes of the reference system with fictitious masses are used for order reduction, a low-frequency subset of the right natural modes $[\Phi_R]$ [see Eq. (4)],

$$[\lambda_i[M + M_{\mathrm{FM}}] + [K]]\{\phi_i\} = \{0\} \tag{31}$$

is used for the direct problem [Eqs. (10–12)], or a subset of the left natural modes $[\Psi_L]$ is used for the adjoint problem [Eqs. (15–17)]

$$\left[\lambda_i[M + M_{\mathrm{FM}}]^T + [K]^T\right]\{\psi_i\} = \{0\} \tag{32}$$

The fictitious mass matrix added to the actual mass matrix is $[M_{\mathrm{FM}}]$

Third, it may be a Ritz vector-order reduction used for either the direct or adjoint models, based on a set of Ritz vectors calculated for the reference structure. Ritz vectors can be fixed or frequency dependent over the desired range of input/response frequencies. If a Ritz vector-order reduction is used for the direct problem (4), then the Ritz vector used is the solution of Eq. (4) corresponding to the force input on the right-hand side. The reduced basis $[\Phi]$ is frequency dependent and consists of a single frequency-dependent vector [approximation (8) and Eqs. (9–12)].

The first three methods all require solutions of a reduced-order problem [approximation (8) and Eqs. (9–12) and approximation (13) and Eqs. (14–17)] for the direct and adjoint problems corresponding to the reduced bases used. Alternatively, first-order approximations for the second-order method can be generated by using the following two solutions.

First is the reference direct solution and/or the reference adjoint solution over all frequencies without change [Eqs. (4–6)]:

$$\{\tilde{u}(j\omega)\} = \{u_{\mathrm{ref}}(j\omega)\} \tag{31}$$

$$\{\tilde{\eta}(j\omega)\} = \{\eta_{\mathrm{ref}}(j\omega)\} \tag{32}$$

Second is Taylor series extrapolations of the reference solutions based on the full-order solution (direct and or adjoint) and sensitivities at the reference design. That is, for the direct problem (4),

$$\{\tilde{u}(j\omega)\} = \{u_{\mathrm{ref}}(j\omega)\} + \sum_i \left.\frac{\partial\{u(j\omega)\}}{\partial \mathrm{DV}_i}\right|_{\mathrm{ref}} (\mathrm{DV}_i - \mathrm{DV}_{\mathrm{ref}i}) \tag{33}$$

and for the adjoint,

$$\{\tilde{\eta}(j\omega)\} = \{\eta_{\mathrm{ref}}(j\omega)\} + \sum_i \left.\frac{\partial\{\eta(j\omega)\}}{\partial \mathrm{DV}_i}\right|_{\mathrm{ref}} (\mathrm{DV}_i - \mathrm{DV}_{\mathrm{ref}i}) \tag{34}$$

Of course, reciprocal approximations[13] can be used in the first-order Taylor series expansions instead of the direct approximations of Eqs. (33) and (34).

In this initial attempt to evaluate second-order analysis and sensitivity approximations for coupled structural–acoustic problem, reduced-order approximations of the coupled problem are used here. That is, when modes are used, these are coupled modes of the structural–acoustic system, and they can be complex. Similarly, when Ritz vectors are used, they consist of responses of the coupled full-order structural–acoustic system to various inputs.

VI. Test Case

Consider the two-dimensional acoustic enclosure surrounded by rigid walls and one flexible wall as shown in Fig. 1. A special purpose two-dimensional finite element capability was created for the analysis of coupled structural–acoustic systems made of structural bar elements and triangular linear-pressure acoustic elements. Concentrated masses and springs are allowed. Structural hysteretic damping $jg[K]\{u\}$ is used to model energy dissipation in the frequency domain. The two-dimensional capability is design oriented.

The model shown in Fig. 1 has 143 acoustic nodes, 44 structural nodes, and a total of 275 degrees of freedom. An additional model, identical in geometry but using a much finer finite element mesh, was also created. This model contained 1147 acoustical nodes, 132 structural nodes, and a total of 1543 degrees of freedom. The models

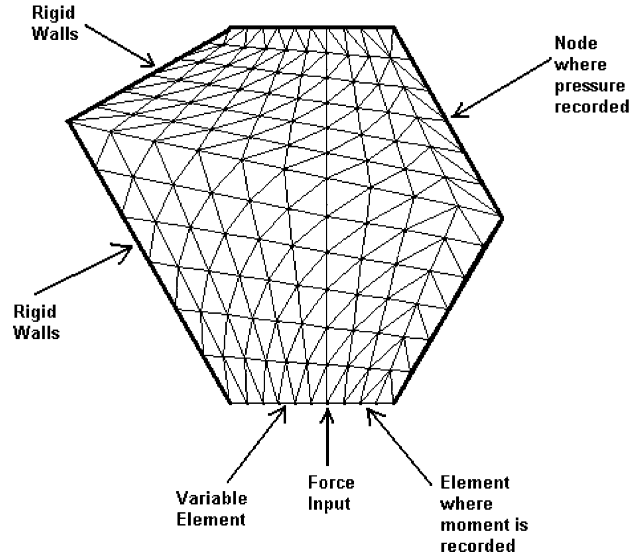

Fig. 1 Two-dimensional structural–acoustic system,[30] small model.

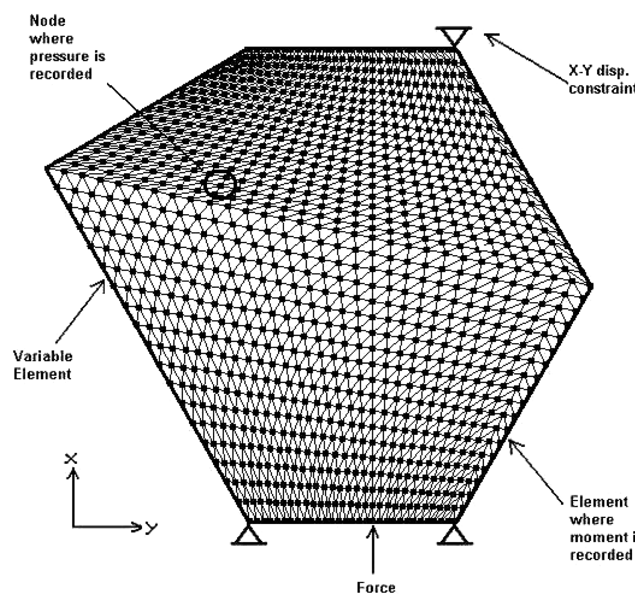

Fig. 2 Two-dimensional structural–acoustic system, large-scale model, 1543 degrees of freedom.

will be referred to as the small- and large-scale models, respectively, throughout this work. (In Fig. 1, rigid walls are shown to simulate the Ref. 30 case. In all sensitivity and approximation test cases studied, all walls are flexible.)

The enclosure shown in Figs. 1 and 2 is 0.508 m wide on the top and bottom and 0.8763 m high. If the lower left corner of the enclosure is labeled vertex 1 and is located at the origin, then going counterclockwise, the coordinates of the vertices are as outlined in Table 1. The beams connecting the vertices and surrounding the enclosure are constrained in the x–y direction at vertices 1, 2, and 4 but otherwise are allowed to rotate freely. All other vertices are allowed to move freely. Also, similar to the model in Ref. 30, two point masses of 0.03 kg were attached to the bottom beam panel at $x = 0.2032$ and 0.3048 m. The excitation force is a point unit load located at $x = 0.3048$ m. The beams surrounding the enclosure are made of aluminum, with modulus of elasticity E of 72 GPa and density of 2600 kg/m^3. The beams have rectangular cross sections with 0.2032 m at the base and height of 0.003175 m. Thus, they have a cross-sectional area of 6.45×10^{-4} m^2 and a moment of inertia of 5.4197×10^{-10} m^4. The fluid in the enclosure is air, with density of 1.225 kg/m^3 and speed of sound of 342 m/s^2.

Table 1 Vertex coordinates of acoustic enclosure

Vertex	x coordinate, m	y coordinate, m
1	0	0
2	0.508	0
3	0.8453	0.5842
4	0.508	1.1684
5	0	1.1684
6	−0.506	0.8763

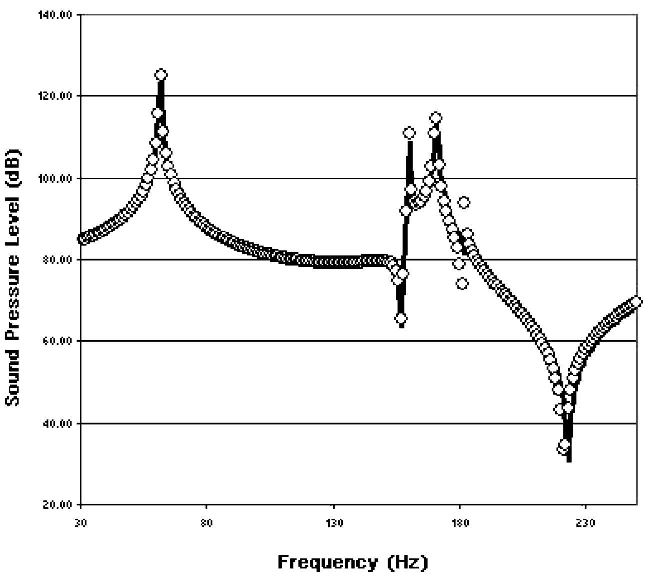

Fig. 3 Frequency-response correlation for Ref. 30 test case; rigid walls except for bottom wall, small model, 275 degrees of freedom: ◇, ANSYS and ——, research code.

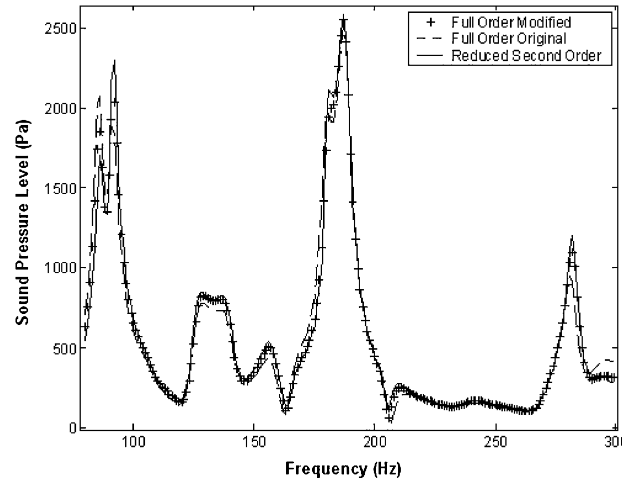

Fig. 4 Pressure response, small model.

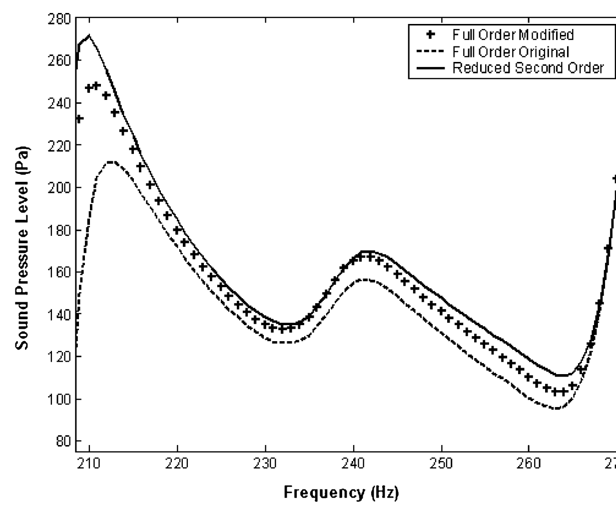

Fig. 5 Pressure response, small model, selected frequency band.

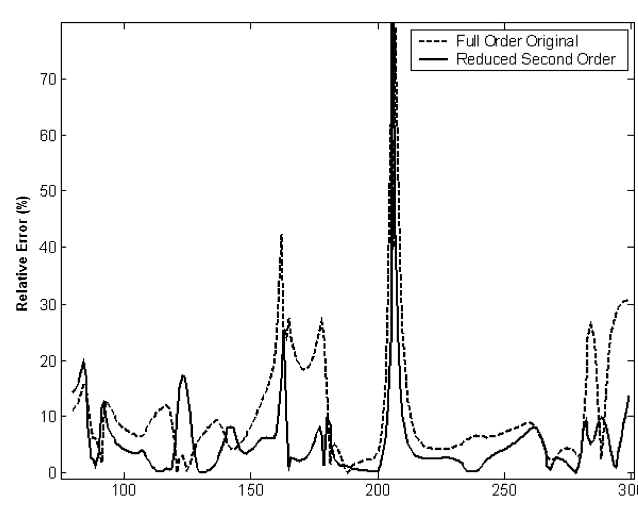

Fig. 6 Relative error (%): pressure response.

In the numerical studies discussed here, the model described was used as a reference model for generating reference direct and adjoint solutions, left and right modes and corresponding natural frequencies, and reference sensitivities. SOA were then compared to first-order approximations for a modified model, where the cross-sectional properties of a variable element were changed. This was done to assess performance of various approximations for possible utilization in an approximation concepts/nonlinear programming-based design optimization strategy,[13] in which the design variables are changed as the model is driven to the optimum. Both the moment of inertia and beam cross-sectional area of the variable element were increased first by 50% and then by 100% where the DV are changed as the model is driven to the optimum.

Results obtained by the present two-dimensional design-oriented coupled structural–acoustic capability used for the studies in this work are compared to results by ANSYS,[31] a standard multiphysics finite element code with structural–acoustic capabilities. The case used for computational verification involved rigid side walls except for the bottom wall. As Fig. 3 shows, the correlation is good. The small finite element model was used for code validation.

VII. Numerical Studies

In the first computational study, direct and adjoint solutions of the reference system are used as approximate direct and adjoint solutions of the modified system, where the cross-sectional properties of the variable element are increased by 100%. There is no computational effort dedicated to any reduced-order direct or reduced-order adjoint solutions, and the reference solutions and reference sensitivities are substituted directly into Eqs. (22) and (27–30).

Figures 4 and 5 show frequency response of pressure (Figs. 1 and 2) with the response over certain frequency ranges enlarged to better distinguish between the performance of different approximations. In Figs. 4–9, the approximate direct and adjoint solutions are the reference direct and adjoint solutions. There is a 100% change in area and moment of inertia of the variable element and $g = 0.06$. Behavior of stress (bending moment) approximations was similar. Figures 6–9 show representative relative errors as functions of frequency for pressure and bending moment responses and their sensitivities.

In the following study of approximation accuracy, the ref. direct solution is used as an approximation for the direct solution of the

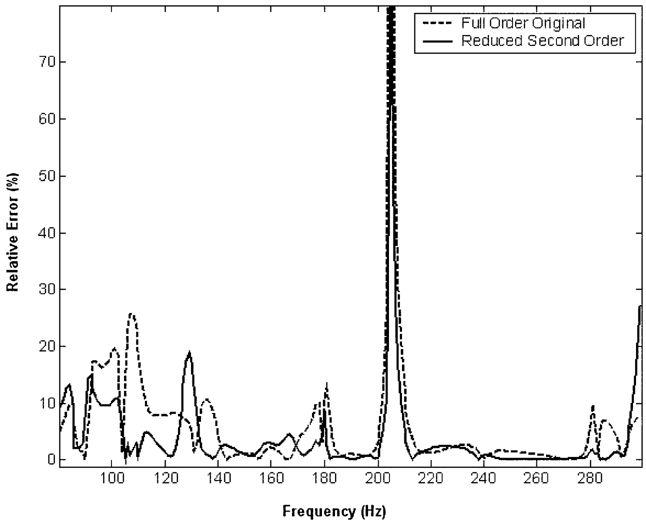

Fig. 7 Relative error (%): bending moment response, small model.

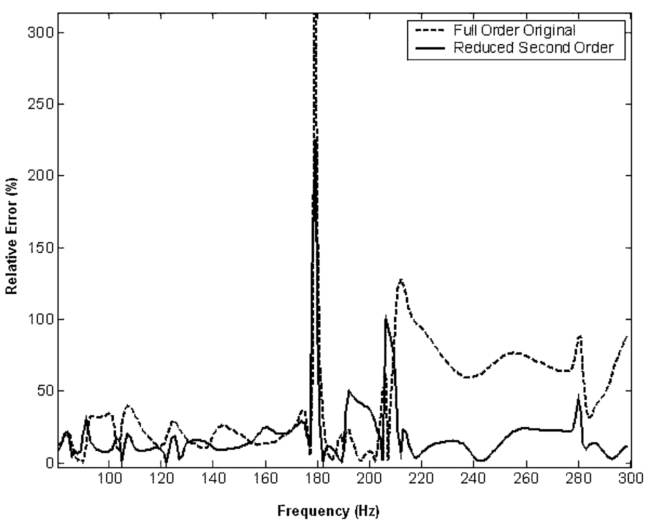

Fig. 8 Error (%) in approximate sensitivity of bending moment response with respect to cross-sectional area of variable element, small model.

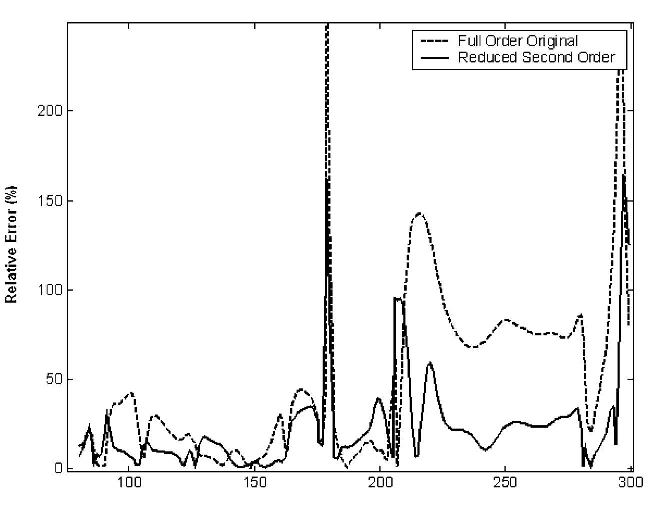

Fig. 9 Error (%) in approximate sensitivity of pressure response with respect to area of variable element, small model.

modified system, where cross-sectional properties of the variable element were increased by 100%. The approximation of the adjoint solution is based on a modal-order reduction of the adjoint problem, using 33 right and left low-frequency eigenvectors of the reference system. Figures 10 and 11 show comparisons of first-order and second-order approximations for pressure and bending moment response over a selected range of frequencies. Errors (percentage) as functions of frequency of excitation (hertz) are shown in Figs. 12–14 for the pressure prediction (where bending moment predictions are similar) and its sensitivity with respect to cross-sectional area of the variable element.

The selection of only 33 low-frequency modes of the ref. structure for modal approximation of the adjoint was done so that the highest frequency in this set of structural–acoustic modes is well below the highest frequency of excitation considered. Indeed, Figs. 12–14 show significant errors in the first-order approximations above about 220 Hz, and the improved accuracy obtained by using the SOA method. Note also the larger errors overall of the approximate sensitivity results compared to analysis results, as expected. In Figs. 10–14, the approximate direct solution is the reference direct solution, and the modified structure analyzed reflects a 100% change in area and moment of inertia of the variable element. The structural damping coefficient is $g = 0.06$. This value was selected to capture high peaks in the response. If g is too low, however, very narrow

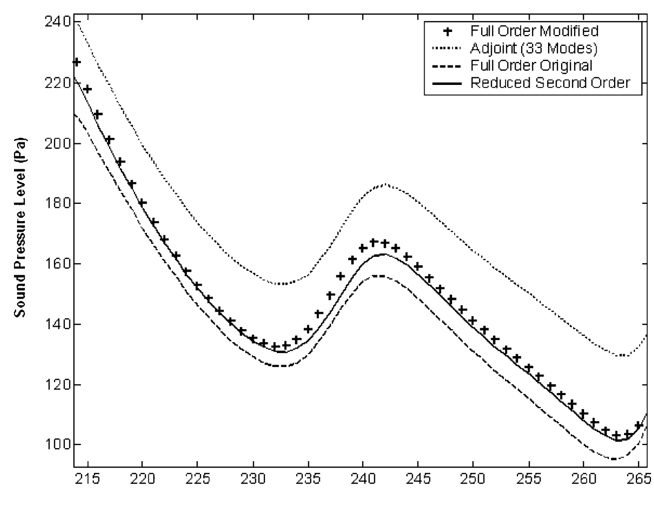

Fig. 10 Pressure response (magnified, over small range of frequencies), small model.

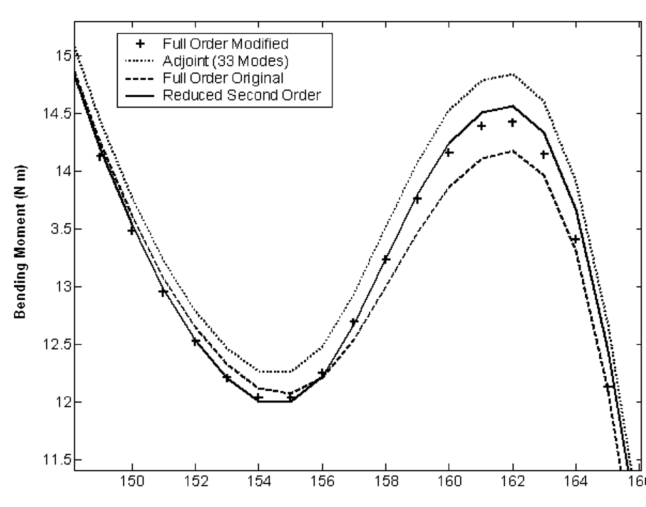

Fig. 11 Bending moment response (magnified, small range of selected frequencies), small model.

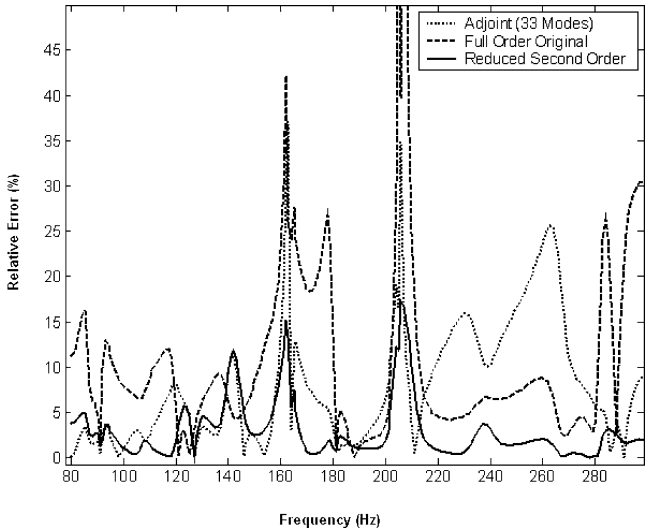

Fig. 12 Pressure approximation error (%), small model.

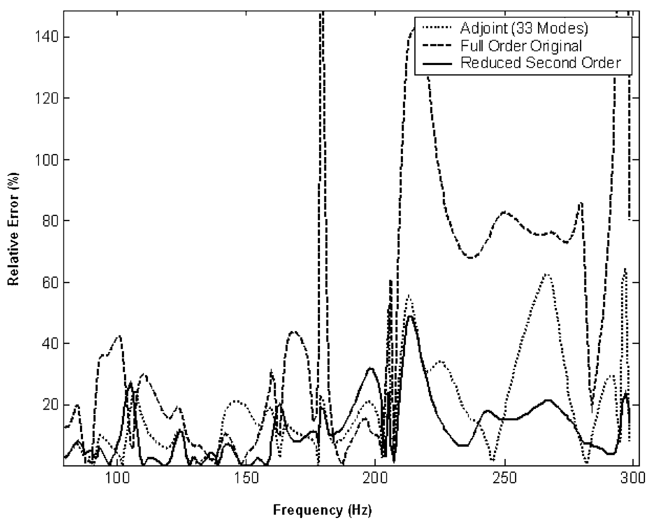

Fig. 13 Error (%) in approximate sensitivity of pressure response with respect to area of variable element, small model.

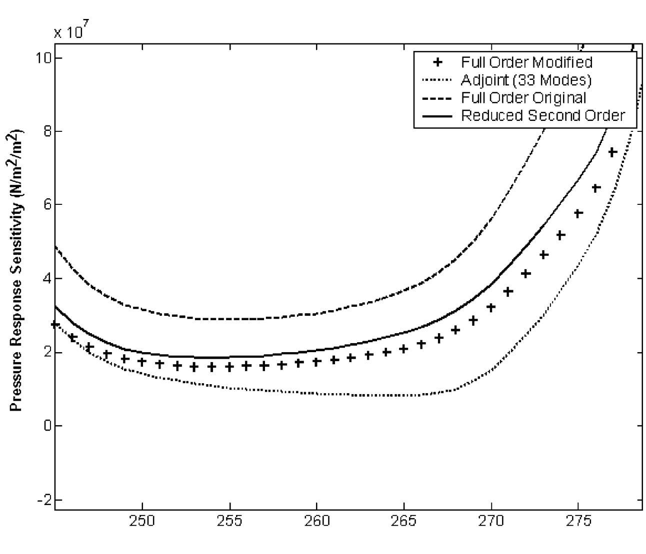

Fig. 14 Full-order and approximate sensitivity (magnified over a selected range of frequencies) of pressure response with respect to area of variable element, small model.

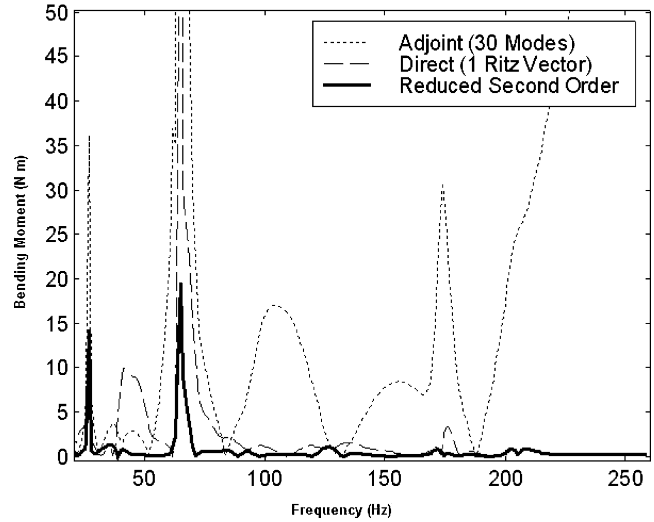

Fig. 15 Errors (%) in bending moment response, large model.

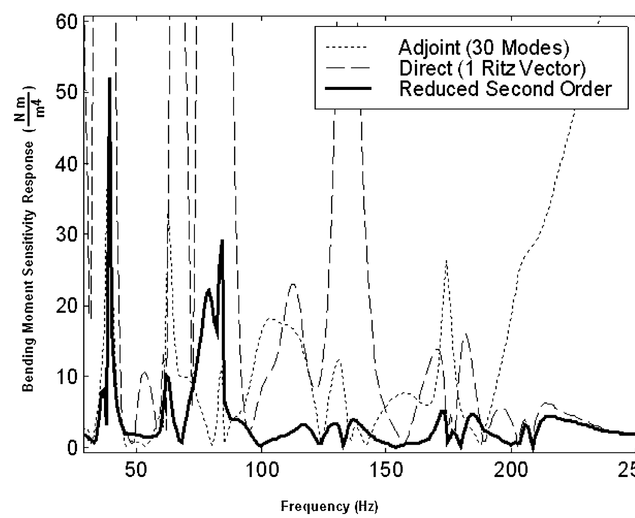

Fig. 16 Errors in bending moment sensitivity with respect to moment of inertia of variable element, large model.

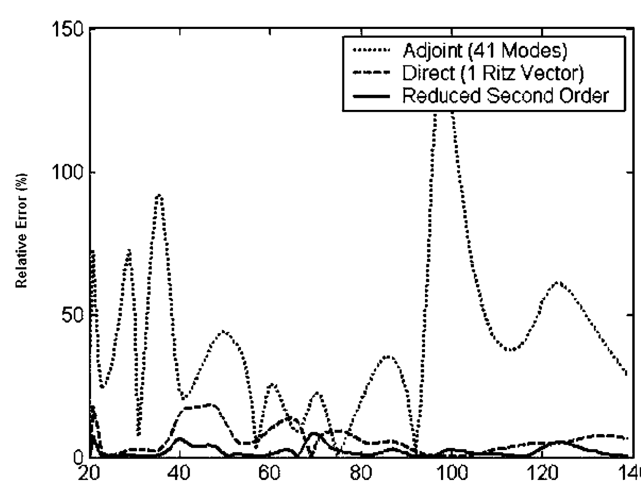

Fig. 17 Error (%) in pressure approximation, small model.

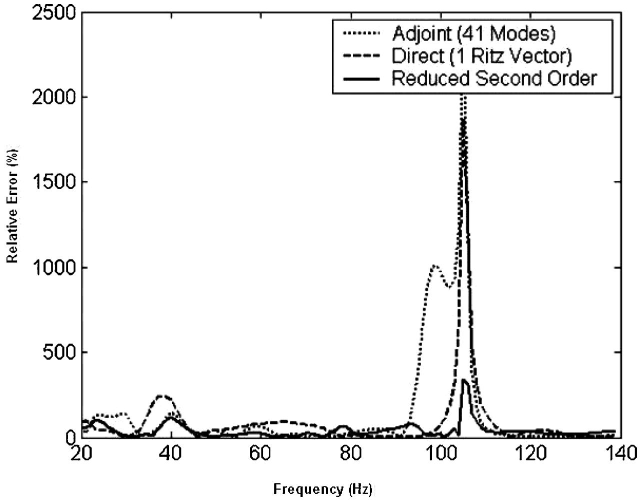

Fig. 18 Error (%) in pressure sensitivity approximation, small model; DV = cross-sectional area.

peaks in the response will lead to large errors on the error vs frequency curves due to even slight shifts in peak frequency, and without comparison of the actual frequency response curves that might be misleading.

When the reference solution (frequency dependent) was used as a Ritz vector to create a 1×1 frequency-dependent Ritz approximation [approximation (13) and Eqs. (14–17)], results of this approximate 1×1 Ritz vector-reduced problem led in general to large inaccuracies. In most cases, the second order method, if the other (complementary) first-order approximation had better accuracy, could improve on the accuracy of both direct and adjoint approximation used. When inaccuracies in either the direct or adjoint approximations were too large, the second-order method usually followed, in terms of accuracy, the more accurate between the two. Figures 15 and 16 show error plots (percent error vs frequency, in hertz) for bending moment and its sensitivities for the large-scale model in the case where a 1×1 Ritz-order reduction [approximation (13) and Eqs. (14–17)] is used for the direct problem and a modal-order reduction with 30 low-frequency modes is used for the adjoint. In Figs. 15 and 16, there is a 100% change in area and moment of inertia of the variable element and $g = 0.06$.

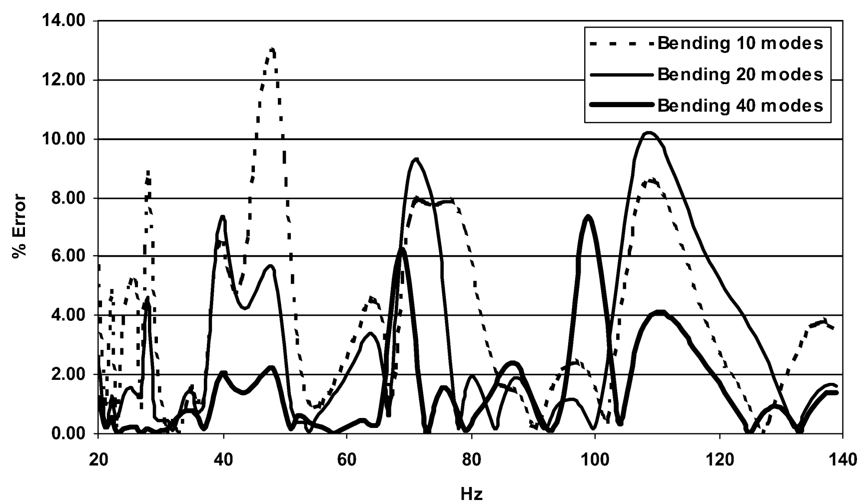

Fig. 19 Convergence (number of modes used): error (%) in bending moment approximation, small model; DV = cross-sectional area.

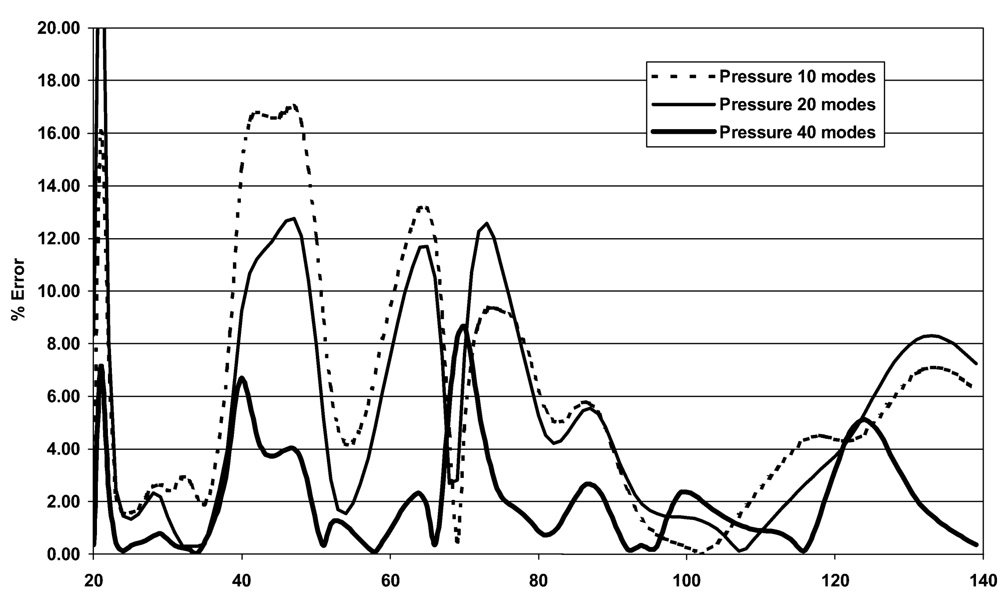

Fig. 20 Convergence (number of modes used): error (%) in pressure approximation, small model; DV = cross-sectional area.

Figures 17 and 18 show accuracy (in terms of percent error vs excitation frequency) when there are significant differences between the ref. structure used to create reduced modal bases and the actual structure these reduced bases are used to solve approximately. In a typical case shown, a direct Ritz approximation, using frequency-dependent solutions for a structure with the variable element at 50% of its final value, are combined with adjoint modal solutions, where the modes used were created for a structure with a much heavier fluid (100 × air density) and much smaller cross sectional areas (1/100 of actual beam areas) of the beam elements. The structure was modified in this way to create in the low-frequency range more modes capable of capturing dynamics of acoustic pressure spatial variation and longitudinal bar element dynamics.

Convergence studies for the SOA are shown in Figs. 19 and 20 for the last case discussed. Note that large error peaks on the error vs frequency curves sometime just reflect a slight shift of the frequency (using the approximate analysis) at which a resonant peak occurs. Figures 19 and 20 show the direct method by 1 × 1 Ritz using direct solutions on a structure with 50% area of the actual variable element. Adjoint approximations are by modal-order reduction using modes created for a structure with 100 × the actual fluid density and 1/100th the actual bars' cross-sectional areas.

Results of additional studies with the SOA are shown in Figs. 21 and 22. The first-order approximations now for both direct and adjoint solutions are Taylor series extrapolations from the reference design (using the reference direct and adjoint solutions and their first-order sensitivities). When first-order Taylor approximations and SOA based on these direct and adjoint approximations are used to predict the frequency response of a system, where the cross section of the variable element is increased by 10%, the second-order method shows improvements in accuracy over the first-order methods for both response and sensitivity of the response at the new design point. Errors grow, however, when direct Taylor series extrapolation is used for the direct and adjoint solutions when the cross-sectional moment of inertia of the variable element is increased by 50%. The SOA method, although improving overall accuracy over some frequency bands, cannot lead to acceptable accuracy when the accuracy of either the direct or adjoint first-order approximations is too poor (Figs. 23 and 24).

Overall, in all of the cases studied, the performance of the SOA was better (on average, over all input frequencies analyzed) than the corresponding first-order approximations, except when one of these first-order approximations was highly inaccurate. Whether first-order approximate analyses are carried out using a reduced base

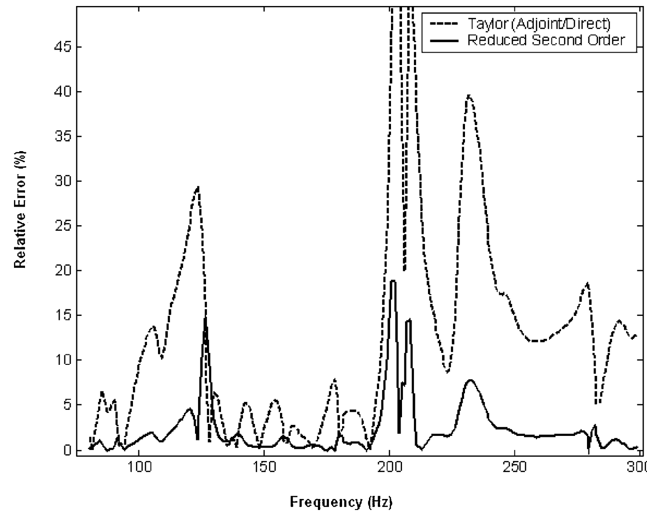

Fig. 22 Errors in first-order Taylor series extrapolations and related SOA for pressure sensitivity with respect to cross-sectional moment of inertia of system with 10% change in variable element cross-sectional properties, small model, 6% structural damping.

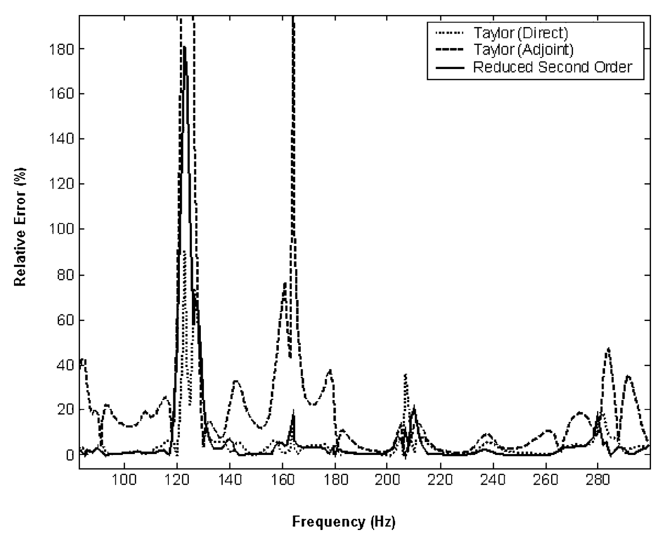

Fig. 23 Errors in first-order Taylor series extrapolations and related SOA for pressure response of system with 50% change in variable element cross-sectional properties, small model, 6% structural damping.

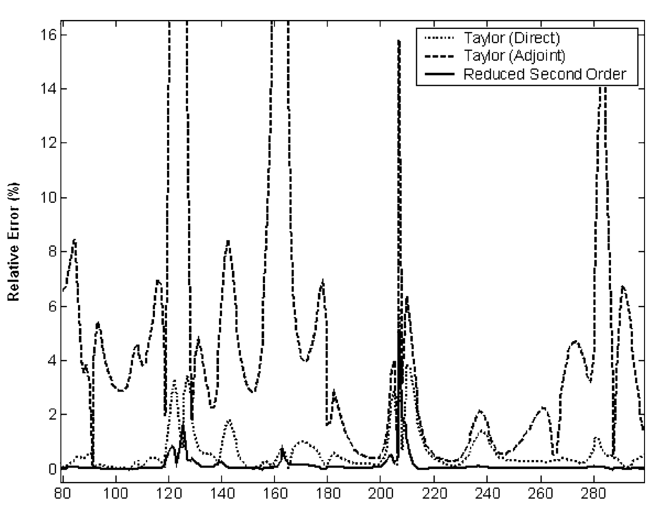

Fig. 21 Errors in first-order Taylor series extrapolations and related SOA for pressure response of system with a 10% change in variable element cross-sectional properties, small model, 6% structural damping.

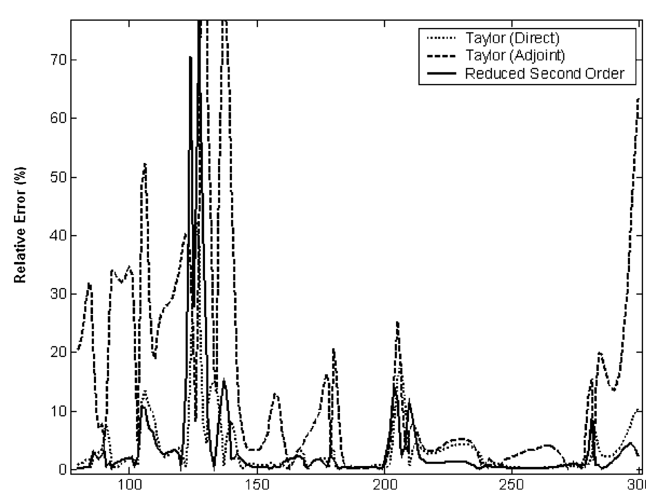

Fig. 24 Errors in first-order Taylor series extrapolations and related SOA for bending moment response of system with 50% change in variable element cross-sectional properties, small model, 6% structural damping.

30–60 of modal vectors, or just the reference direct and/or adjoint solution, or Taylor series extrapolations based on reference direct and adjoint solutions and their sensitivities, the approximations discussed here lead to significant computational savings. Such savings can be materialized by replacing a full-order structural dynamics problem of the order of 100,000s of degrees of freedom that has to be evaluated at hundreds of frequencies by an approximate SOA based on the reference solutions, or Taylor series extrapolation of the ref. solutions, or combinations of reference and Ritz solutions with modal solutions with up to 50 modes or so (and the corresponding 50×50 reduced-order problems), and using such reduced-order modeling techniques coupled structural–acoustic frequency domain optimization can become more feasible.

VIII. Conclusions

Representative results of approximation accuracy studies were presented using the combined direct/adjoint SOA method to approximate frequency-response solutions for a typical structural acoustic problem involving an acoustic enclosure surrounded by flexible walls. Application of the combined direct/adjoint second-order method was extended from structural dynamics and unsteady aerodynamics to coupled structural acoustic. A variety of first-order approximations were used for the reduced-order direct and adjoint problems. Bending moment (stress) and acoustic pressure responses and their sensitivities with respect to cross-sectional properties of structural wall elements were studied, as well as approximation accuracy when reduced-order bases generated for a reference system are used to reduce the order of a system that evolved and was considerably modified relative to the reference design due to design variations in the course of an optimization process.

It was found that the second-order method could improve accuracy of approximation over first-order methods, provided that there was no major inaccuracy in either the direct or adjoint first-order approximation used. In some cases, SOA led to accurate results even in the presence of large errors in the first-order approximations used, for example, in cases when not enough modes were retained in the modal base to cover the range of excitation frequencies considered. However, this result was not general and was case dependent. Second-order combined direct/adjoint order-reduction approximations, when improving the accuracy provided by first-order approximations, lead to major computational savings. In the case of structural acoustics in the frequency domain involving very large coupled structures–acoustics finite element models, effective use of the combined direct/adjoint second-order method can make an intractable design optimization problem into a practical one, when a strategy of approximation concepts/nonlinear programming is used.

Acknowledgment

Support for this research by the U.S. Air Force Office of Scientific Research is gratefully acknowledged.

References

[1]Everstine, G. C., "Finite Element Formulations of Structural Acoustics Problems," *Computers and Structures*, Vol. 65, No. 3, 1997, pp. 307–321.

[2]Kardestuncer, H., and Norrie, D. H., *Finite Element Handbook*, McGraw–Hill, 1987, Sec. 7.4.

[3]Zienkiewicz, O. C., and Taylor, R. L., *The Finite Element Method*, 4th ed., Vol. 2, 1991, McGraw–Hill, London, Chap. 11.

[4]Nefske, D. J., and Sung, S. H., "Vehicle Interior Acoustic Design Using Finite Element Methods," *International Journal of Vehicle Design*, Vol. 6, No. 1, 1985, pp. 24–40.

[5]Sung, S. H., and Nefske, D. J., "A Coupled Structural–Acoustic Finite Element Model for Vehicle Interior Noise Analysis," *Journal of Vibration, Acoustics, Stress and Reliability in Design*, Vol. 106, 1984, pp. 314–318.

[6]Choi, K. K., Shim, I., Lee, J., and Kulkami, H. T., "Design Sensitivity Analysis of Dynamic Frequency Responses of Acousto-Elastic Built-Up Structures," *Optimization of Large Structural Systems*, edited by G. I. N. Rozvany, Vol. 1, Kluwer Academic, Dordrecht, The Netherlands, 1993, pp. 329–343.

[7]Wenigwieser, C., and Haeusler, S., "Finite Element Application to Interior Noise Prediction in Aircraft Fuselage," *Proceeding of the 17th Congress of the International Council of the Aeronautical Sciences*, 1990, pp. 2094–2104.

[8]Morand, H. J.-P., and Ohayon, R., *Fluid–Structure Interaction: Applied Numerical Methods*, Wiley, 1995.

[9]Ohayon, R., and Soize, C., *Structural Acoustics and Vibrations*, Academic Press, San Diego, CA, 1998.

[10]Ihlenburg, F., *Finite Element Analysis of Acoustic Scattering*, Springer-Verlag, New York, 1998.

[11]Habault, D., (ed.), *Fluid–Structure Interactions in Acoustics*, Springer-Verlag, Vienna, 1999.

[12]Johnson, E. H., "Adjoint Sensitivity Analysis in MSC/NASTRAN," MSC NASTRAN World Users Conf. 1997.

[13]Haftka, R. T., and Gurdal, Z., *Elements of Structural Optimization*, 3 rev. and expanded addition, Kluwer Academic, Dordrecht, The Netherlands, 1992.

[14]Barthelemy, J.-F. M., and Haftka, R. T., "Approximation Concepts for Optimum Structural Design—A Review," *Structural Optimization*, Vol. 5, 1993, pp. 129–144.

[15]Li, W.- L., and Livne, E., "Analytic Sensitivities and Approximations in Supersonic and Subsonic Wing/Control Surface Unsteady Aerodynamics," *Journal of Aircraft*, Vol. 34, No. 3, 1997, pp. 370–379.

[16]Livne, E., and Blando, G. D., "Reduced-Order Design-Oriented Stress Analysis Using Combined Direct and Adjoint Solutions," *AIAA Journal*, Vol. 38, No. 5, 2000, pp. 898–909.

[17]Livne, E., and Blando, G. D., "Structural Dynamic Frequency Response Using Combined Direct and Adjoint Reduced-Order Approximation," *AIAA Journal*, Vol. 41, No. 7, 2003, pp. 1377–1385.

[18]Van-Niekerk, B., "Computation of Second-Order Accurate Unsteady Aerodynamic Generalized Forces," *AIAA Journal*, Vol. 24, No. 3, 1986, pp. 492–498.

[19]Canfield, R. A., "Design of Frames Against Buckling Using a Rayleigh Quotient Approximation," *AIAA Journal*, Vol. 31, No. 6, 1993, pp. 1143–1149.

[20]Craig, R. R., *Structural Dynamics*, Wiley, New York, 1981.

[21]Kline, K. A., "Dynamic Analysis Using a Reduced Basis of Exact Modes and Ritz Vectors," *AIAA Journal*, Vol. 24, No. 12, 1986, pp. 2022–2029.

[22]Wilson, E. L., "A New Method of Dynamic Analysis for Linear and Non-Linear Systems," *Finite Element in Analysis and Design*, Vol. 1, 1985, pp. 21–23.

[23]Arnold, R. R., Citerley, R. L., Chargin, M., and Galant, D., "Application of Ritz Vectors for Dynamic Analysis of Large Structures," *Computers and Structures*, Vol. 21, No. 3, 1985, pp. 461–467.

[24]Livne, E., "Accurate Calculation of Control Augmented Structural Eigenvalue Sensitivities Using Reduced-Order Models," *AIAA Journal*, Vol. 27, No. 7, 1989, pp. 947–954.

[25]Sandridge, C. A., and Haftka, R. T., "Accuracy of Eigenvalue Derivatives from Reduced-Order Structural Models," *Journal of Guidance, Control, and Dynamics*, Vol. 12, No. 6, 1989, pp. 822–829.

[26]Sandridge, C. A., and Haftka, R. T., "Modal Truncation, Ritz Vectors, and Damping-Ratio Derivatives in Structural Control," *Journal of Guidance, Control, and Dynamics*, Vol. 14, No. 4, 1991, pp. 785–790.

[27]Karpel, M., and Raveh, D., "Fictitious Mass Element in Structural Dynamics," *AIAA Journal*, Vol. 34, No. 3, 1996, pp. 607–613.

[28]Morand, H., and Ohayon, R., "Substructure Variational Analysis of the Vibrations of Coupled Fluid–Structure Systems. Finite Element Results," *International Journal for Numerical Methods in Engineering*, Vol. 14, 1979, pp. 741–755.

[29]Wang, X., "On a Mode Superposition Method for Fluid–Structure Interaction Problems," *Dynamics, Acoustics, and Simulations*, ASME DE-Vol. 98, American Society of Mechanical Engineers, 1998, pp. 71–77.

[30]Unruh, J. F., "Finite Element Subvolume Technique for Structural-Borne Interior Noise Prediction," *Journal of Aircraft*, Vol. 17, No. 6, 1980, pp. 434–441.

[31]Anysys, Inc., *Theory Reference*, Canonsburg, PA, 1999, Chap. 8.

C. Pierre
Associate Editor